

INFLATION IN S-DUAL SUPERSTRING MODELS

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ABSTRACT

We study inflationary potentials in the framework of superstring theories. Successful inflation may occur due to chiral fields, but only after the dilaton and moduli are stabilized. This is achieved by demanding an S-duality invariant potential. Then, it is possible to have inflation at any scale and even to have more than one stages of inflation. This occurs for a limited class of scalar potentials, where certain conditions are obeyed. Density fluctuations of 10^{-5} require the inflationary potential to be at a scale of $V^{1/4} \simeq 5 \times 10^{15}$ GeV.

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Inflationary scenarios [1] address several of the problems of the standard hot big-bang theory, such as the flatness and horizon problems, the overabundance of topological defects that result from the breaking of a GUT symmetry and the origin of density fluctuations in the universe [2]. Among the several possibilities, one of particular interest is chaotic inflation, which provides a natural set of initial conditions for the inflaton field [3]. In this scheme however, in order to obtain sufficient growth of the scale factor as well as the correct magnitude of density fluctuations, the introduction of an arbitrary, tiny coupling constant, associated with the interaction strength in the scalar sector is required. This is avoided in theories where a transition from a higher dimensional to a four-dimensional universe occurs [4], since they are governed by only one scale, the Planck mass M_{Planck} . The best candidate of a higher dimensional unification is superstring theory and there are several proposals on inflation from superstrings [5]-[10].

In string potentials, inflation may occur only after freezing the dilaton field, and before this happens no other field may be used as the inflaton. Indeed, in schemes where inflation is due to the scalar potential, it is not possible to obtain enough e-folds of inflation while the dilaton is a dynamical field [11], even when the loop effects from the 4-Fermi gaugino interactions are included [10]. (In an alternative solution, inflation occurs due to the kinetic energy of the inflaton [9]). The stabilization of the dilaton field has many important phenomenological consequences. For example, it fixes the gauge coupling constant at the string scale and sets the hierarchy between the masses of the fermions and their supersymmetric partners. Even though the vev (vacuum expectation value) of the dilaton is essential in understanding the low energy models of string theory, there is still no definite answer as to how it arises. A plausible way to fix the vev of the dilaton is via gaugino condensates [12],[13], which are expected to form if the string model has an asymptotically free gauge group. In [10] we studied the inflationary potentials in the presence of gaugino condensates. Even though enough inflation can be obtained to solve the flatness and horizon problems, the scale at which the dilaton freezes is in these schemes 10^{12-13} GeV². The density fluctuations generated by inflation at or below the condensation scale are then much smaller [10] than the observed by COBE [14].

Here, we will study the possibility of having inflationary potentials above the supersymmetry breaking scale, but still demanding a stable dilaton field. An interesting possibility to stabilize the dilaton at a higher scale appears when an S-duality symmetry of the lagrangian (conjectured by Ibanez et al. in the framework of superstring theories) is considered [15]. A great deal of work has been invested in studying this symmetry and related ones, due to the results of Seiberg and Witten [16], which were obtained for $N = 2$ SUSY. It is still not clear whether S-duality will survive in an $N = 1$ SUSY theory. However, we think that it is interesting to study the consequences for inflation, assuming that such a symmetry remains valid in $N = 1$ SUSY lagrangians. Even more so, since there are no other alternatives to fix the dilaton field at a large scale of 10^{15} GeV that we know of. Demanding an S-dual potential, inflation may occur at a higher scale than the condensation scale, resulting to larger density fluctuations. We will show that an S-dual potential allows to have inflation at any scale

²This scale is obtained by demanding that the supersymmetric partners of the fermions have masses around 1 TeV, as required by supersymmetry breaking arguments.

and also opens the possibility of having more than one stages of inflation in string models. Normalization of the density fluctuations with respect to COBE requires a spontaneously broken symmetry at around $V_{Inf}^{1/4} \simeq 5 \times 10^{15} GeV$. The breaking of symmetries is expected in string models since the structure is very rich [7] and one expects, in general, scalar fields to acquire a vev either at tree level or via radiative corrections as in the breaking of the electroweak symmetry.

To see how this works, we impose S-duality to the low energy superstring Lagrangian. The simplest realization is to take S-duality as an $SL(2, \mathbb{Z})$ symmetry generated by $S \rightarrow 1/S$ and $S \rightarrow S + i$ (this last symmetry is already present in all 4-D string vacua). In order to achieve the $SL(2, \mathbb{Z})$ invariance one can distinguish two different cases [17]: (1) $f \rightarrow f$ or (2) $f \rightarrow 1/f$, where f is the gauge kinetic function. In the first case the gauge coupling constant, given by $g^2 = Re f$, remains invariant and the gauge chiral superfield W_α coupled to f by a term $(f W_\alpha W^\alpha)_F + h.c$ will also be invariant. On the other hand, in case (2) one has $g^2 \rightarrow 1/g^2$ and the gauge chiral superfield is no longer invariant and one requires additional fields called "magnetic mesons" [15] to fully realized the symmetry. It is not clear whether an $N=1$ supergravity is reliable in this case. The gauge kinetic function can be written in case (1) as $f = (1/2\pi) \ln(j(S) - 744)$ [17], where $j(S)$ is the generator of modular invariant functions³. This f reproduces the large S limit calculated perturbatively, i.e. $f = S$. We are interested in the scalar sector only and will therefore not consider the gauge sector of the theory. Since the gauge coupling constant enters in the scalar sector via the D -term with $V = \frac{1}{2Re f} D^2$ then for $D = 0$ the gauge and scalar sectors decouple.

The effective $d = 4$ superstring model is given by an $N = 1$ supergravity theory [18] with at least four gauge singlet fields S and T_i , $i = 1, 2, 3$ as well as an unspecified number of gauge chiral matter superfields. The tree level scalar potential is given by [18]

$$V = e^K H + \frac{1}{2Re f} D^2 \quad (1)$$

$$\begin{aligned} H &= (G_a (K^{-1})^a_b G^b - 3|W|^2) \\ D &= \hat{g} K^i T_i^j \phi_j + \xi \end{aligned} \quad (2)$$

where D is the auxiliary field of the gauge vector superfield, T_i^j the generators of the gauge group, \hat{g} the charge, ξ a Fayet-Illiopoulos term and

$$\begin{aligned} G &\equiv K + \ln|W|^2 \\ K &= -\log(S_r) - \log \left[(T_r - \phi \bar{\phi})^3 - B \bar{B} - T_r (C \bar{C}) \right] \\ W &= \Omega(S) P(T, \phi, B, C) \\ \Omega(S) &= \eta^{-2}(S) \end{aligned} \quad (3)$$

Here G is the Kähler potential, $T_r = T + \bar{T}$, $S_r = S + \bar{S}$ and the indices a, b run over all fields, i.e. the dilaton S , the moduli T and chiral fields ϕ , B and C . The ϕ fields correspond to untwisted chiral fields while B and C are twisted fields. The indices a, b of the functions K and W denote derivatives with respect to chiral fields [19]. All the fields are normalized

³For large S one has $j(S) = 1/q + 744 + 196884 q + O(q^2)$, $q = e^{-2\pi S}$.

with respect to the reduced Planck mass $m_p = M_p/\sqrt{8\pi}$. The form of K is derived by a perturbative expansion and is valid if the arguments inside the logarithms are positive. The term $\Omega(S)$ (where η is the Dedekind-eta function with modular weight $1/2$) arises due to S-duality in order to cancel the transformation of K and to render $G = K + \ln|W|^2$ invariant. A similar situation appears for the moduli fields T_i . The function $\Omega(S)$ may be multiplied by a modular invariant function but, for simplicity, we have chosen it to be unity. The chiral superpotential P is a polynomial function of the chiral fields and in particular, it contains the trilinear (Yukawa) interactions of the chiral fields. Here, we work with the effective low energy superstring Lagrangian in the Einstein and not in the Brans-Dicke frame [20], which would seem more natural in the context of string inflation. This is for continuity, since most of the work in determining the non-perturbative contributions of S and T to the potential as well as the study of the duality symmetries for T and S , has been carried out in the Einstein frame.

From now on we work with the untwisted fields ϕ , which we found in [10] to be best suited for inflation than B and C , due to the form of K . Considering only untwisted fields in the sector related to T_1 , the superpotential can be written as $W = \eta^{-2}(S)\eta^{-2}(T_2)\eta^{-2}(T_3)P(T_1, \phi)$. Here we take P in the form $P(T_1, \phi) = \lambda(T_1)P(\phi)$. The derivatives of the Kähler potential are

$$\begin{aligned} G_S(K^{-1})^S_S G^S &= |W|^2 \frac{S_r^2}{4\pi^2} |\hat{G}_2(S)|^2 \\ G_{T_i}(K^{-1})^{T_i}_{T_i} G^{T_i} &= |W|^2 \frac{T_{ri}^2}{4\pi^2} |\hat{G}_2(T_i)|^2 \\ G_m(K^{-1})^m_n G^n &= |W|^2 + W_m(K^{-1})^m_n W^n + (K_m W(K^{-1})^m_{T_1} W^{T_1} + h.c.) \end{aligned} \quad (4)$$

Here, $T_i, i = 2, 3$ and m, n run over T_1 and the untwisted chiral fields related to the T_1 sector. Using eqs.(3) and (5) the scalar potential is given by

$$\begin{aligned} V = e^K |\eta(T_2)\eta(T_3)\eta(S)|^{-4} &\left(|P|^2 \left[\frac{S_r^2}{4\pi^2} |\hat{G}_2(S)|^2 + \sum_i \frac{T_{ri}^2}{4\pi^2} |\hat{G}_2(T_i)|^2 - 2 \right] \right. \\ &\left. + P_m(K^{-1})^m_n P^n + (K_m P(K^{-1})^m_{T_1} P^{T_1} + h.c.) \right) \end{aligned} \quad (5)$$

where $\hat{G}_2(S)$ is the Eisenstein function of modular weight 2. The potential in eq.(5) is manifestly S-duality invariant (and $T_{2,3}$ -duality invariant) since all the dependence on S is given in terms of duality invariant functions $e^K |\eta(S)|^{-4} \simeq (S_r |\eta(S)|^4)^{-1}$ and $S_r^2 |\hat{G}_2(S)|^2$ (note that P and its derivatives are S and T_i independent). Under quite general initial conditions the term $P_m(K^{-1})^m_n P^n + (K_m P(K^{-1})^m_{T_1} P^{T_1} + h.c.)$ will give a positive vacuum potential. If V is positive then, independently of P and P_m , minimization of V_0 with respect to S, T_i gives $\langle S \rangle = 1, e^{-\pi/6}$ and $\langle T_i \rangle = 1, e^{-\pi/6}$ (the dual invariant points) where $S_r^2 |\hat{G}_2(S)|^2 = T_r^2 |\hat{G}_2(T)|^2 = 0$ and $(S_r |\eta(S)|^4)^{-1} \approx 1$ take its minimum value. We will consider models where the D term vanishes and thus the S dependence on Ref is irrelevant. We also assume a $\lambda(T_1)$ which leads to a vev unity for T_1 as well.

Having the dilaton and the moduli frozen, one may then look whether inflation occurs due to chiral fields. In order to have inflation, the potential V may not be dominated by the terms proportional to P (implying certain cancellation conditions) [5],[10], while to have

a large number of e-folds the vev of the inflaton field must be of the order of the Planck mass. We look for solutions where the scale of inflation is below the v.e.v of the higgs boson that breaks the gauge group to the standard model group. Therefore, we take the inflaton superpotential as $P = \Phi^2 F(\phi_i)$ where Φ is a bosonic field that acquires a vev below the unification scale and $F(\phi_i)$ is a function of chiral fields containing the inflaton field. The requirements for inflation on F are: $F \ll F_i$ and $F_{ii} < F_i$ where $F_i = \partial F / \partial \phi_i$. We also include a D term for Φ . This term gives a vev to Φ at a scale M . The appearance of scalar fields with nonvanishing v.e.vs is to be expected in string and supergravity theories, since a symmetry breaking below the Planck scale in a sector of the theory introduces masses to all sectors [7].

A simple example is given by the superpotential

$$\begin{aligned} P &= \lambda(T_1) \Phi^2 F \\ F &= \phi_1^2 - \phi_2^2 \end{aligned} \quad (6)$$

and a D -term

$$D^2 = \hat{g}^2 (|\Phi|^2 - M^2)^2 \quad (7)$$

where λ is the T_1 -dependent Yukawa coupling, \hat{g} the charge of Φ (which we will take as one, i.e. $\hat{g} = 1$) and M the gauge unification scale. As mentioned above we take $\lambda(T_1)$ to be such that the vev of $T_1 = 1$ and $\lambda = 1$. This superpotential satisfies the conditions $F \ll F_i$ and $F_{ii} < F_i$ if $\phi_1 \simeq \phi_2$. We will show that these conditions are actually dynamically satisfied since minimization with respect to ϕ_i gives $\phi_1 \simeq \phi_2$. The field Φ is already at its minimum (and cannot be used as the inflaton field) and its vev, given in terms of M , will fix the magnitude of the fluctuations. We note that if instead of the above superpotential a P trilinear in the untwisted fields is used, it is not obvious that one can have a proper vev for Φ^4 , satisfy the cancellation conditions and achieve an end to inflation. The absence of trilinear terms can be explained by imposing an R-symmetry invariance [7]-[8] which forbids them.

Since the vev of Φ is much smaller than one, the normalization factor of the field is not relevant. Then, in order to simplify the presentation we consider the Φ field to be canonically normalized. We take $\phi_i, i = 1, 2$ to be untwisted chiral fields belonging to one sector of the orbifold only (the sector associated with T_1). The modular weights of these fields will be different than zero only with respect to T_1 and the Kähler potential is

$$\begin{aligned} K &= K_0 + |\Phi|^2 - \ln Q \\ Q &\equiv T_{r_1} - |\phi_1|^2 - |\phi_2|^2 \\ K_0 &= -\ln(S + \bar{S}) - \ln(T_2 + \bar{T}_2) - \ln(T_3 + \bar{T}_3) \end{aligned} \quad (8)$$

The superpotential and Kähler potential of eq.(6) and (8) will inflate. The energy scale will be set by the value of $\langle \Phi \rangle$ which is dynamically determined by $\langle \Phi \rangle \simeq M$. Demanding

⁴A Φ^2 term is needed in order to obtain the correct magnitude of density fluctuations with a field vev of the order of magnitude of $V^{1/4}$ as expected from general spontaneously symmetry breaking arguments.

that the density fluctuations coincide with the ones observed by COBE gives the required value of $\langle \Phi \rangle$. The vev of Φ provides for the explanation of the smallness of the fluctuations, since now the factor $\lambda(T)$ is of order unity, unlike the case discussed in [10] where the dilaton and moduli fields are minimized due to the appearance of a gaugino condensate and no other scale needs to be introduced.

For inflation to occur in supergravity models, some cancelation must occur between the terms $K'H$ and H' of the derivative of the potential, $V' = e^K(K'H + H')$ with $V = e^K H$, with respect to the inflaton field [5],[10]. This is achieved in our example since $\frac{\partial V}{\partial \phi_2} = 0$ implies that $\langle \phi_2 \rangle = \langle \phi_1 \rangle$ and $P = 0$. Note that we cannot have $G_1 = K_1 P + P_1 = \lambda(T_1)\Phi^2(\bar{\phi}_1(\phi_1^2 - \phi_2^2)/Q + 2) = 0$

$$\text{and } G_2 = K_2 P + P_2 = \lambda(T_1)\Phi^2(\bar{\phi}_2(\phi_1^2 - \phi_2^2)/Q - 2) = 0$$

at the same time. For $\phi_1 = \pm \phi_2$ we have $|G_1| = |G_2|$ and $|G_{1,2}|_{\phi_1=\phi_2} \leq |G_{1,2}|_{\phi_1=-\phi_2}$. Of course ϕ_2 will not be identical to ϕ_1 . In particular during inflation we know that the fluctuations of a canonically normalized field are given by the Hawking temperature $\delta\phi = H/2\pi$ which gives $|\langle \phi_2 \rangle - \langle \phi_1 \rangle| \geq H/2\pi$ (with H the Hubble constant). However, to a first approximation we can take $\langle \phi_2 \rangle = \langle \phi_1 \rangle$.

Using eqs.(5), (8) and (7) the scalar potential is simply given by

$$\begin{aligned} V &= \frac{e^{|\Phi|^2}}{Q} \left(Q|G_1|^2 + Q|G_2|^2 + |G_\Phi|^2 + 2Q|G_T|^2 + Q[G_T(G_1\phi_1 + G_2\phi_2) + h.c.] - 3|W|^2 \right) + \\ &+ \frac{1}{2}(|\Phi|^2 - M^2)^2 \\ V &= \frac{e^{|\Phi|^2}}{Q} A \left(Q|\frac{\bar{\phi}_1}{Q}P + P_1|^2 + Q|\frac{\bar{\phi}_2}{Q}P + P_2|^2 + |P|^2 \left[|\bar{\Phi} + \frac{2}{\Phi}|^2 + 2Q - \frac{1}{Q} + a|^2 - 3 \right] \right) + \\ &+ \left(Q\bar{P}(-\frac{1}{Q} + a) \left[(\frac{\bar{\phi}_1}{Q}P + P_1)\phi_1 + (\frac{\bar{\phi}_2}{Q}P + P_2)\phi_2 \right] + h.c. \right) + \frac{1}{2}(|\Phi|^2 - M^2)^2 \end{aligned} \quad (9)$$

where we included the $S, T_i, i = 2, 3$ contributions into $A \equiv e^{K_0}|\eta(T_2)\eta(T_3)\eta(S)|^{-4}$ and we used $a \equiv P_{T_1}/P$, $\hat{g} = 1$, $P_1 = -P_2$. Note that the potential in eq.(9) is semi-positive definite since $|\bar{\Phi} + \frac{2}{\Phi}|^2 - 3 > 0$. Therefore, the minimum is at $S = T_i = 1$ giving $A \simeq 1$ and $\langle \text{Ref} \rangle = 1$. Minimizing with respect to ϕ_2 gives $\phi_2 = \phi_1$ or $P = F = 0$ and eq.(9) becomes

$$V = \frac{e^{|\Phi|^2}}{Q} A 2(K^{-1})_1^1 |P_1|^2 + O(P) + \frac{1}{2}(|\Phi|^2 - M^2)^2 \quad (10)$$

The derivative of the potential with respect to ϕ_1 (using $(K^{-1})_1^1 = Q$) is

$$V_1 = \frac{e^{|\Phi|^2}}{Q} A (2P_1 P^1 (K_1 (K^{-1})_1^1 + (K^{-1})_{11}^1) + (K^{-1})_1^1 \bar{P}^1 P_{11}) + O(P) \quad (11)$$

Note that the term proportional to P_1^2 in V_1 cancels⁵ (i.e. $P_1 P^1 (K_1 (K^{-1})_1^1 + (K^{-1})_{11}^1) = 0$). This cancelation can be traced to the form of K for untwisted fields (cf. eq.(8)) and is

⁵Here $K_i = \frac{\partial K}{\partial \phi_i} = \frac{\bar{\phi}_i}{Q}$, $K_{T_1} = -\frac{1}{Q}$.

fundamental in order to have an inflationary potential. Using eq.(6) and taking $A = 1$ one can write (9) and (11) as $V = e^{|\Phi|^2} |\Phi|^4 2|F_1|^2 + \frac{1}{2}(|\Phi|^2 - M^2)^2$,

$$V = e^{|\Phi|^2} |\Phi|^4 8|\phi_1|^2 + \frac{1}{2}(|\Phi|^2 - M^2)^2 \quad (12)$$

and the derivative with respect to ϕ_1 is $V_1 = e^{|\Phi|^2} |\Phi|^4 F^1 F_{11}$,

$$V_1 = e^{|\Phi|^2} |\Phi|^4 16\bar{\Phi}. \quad (13)$$

The extremum condition $V_\Phi = 0$ gives

$$\langle |\Phi|^2 \rangle = \frac{M^2}{1 + 8|\phi_1|^2} \quad (14)$$

and a scalar potential

$$V = \frac{8M^4|\phi_1|^2}{1 + 16|\phi_1|^2} \quad (15)$$

where we have set $e^{|\Phi|^2} = 1$ since $|\langle \Phi \rangle| \ll 1$. The scalar potential V vanishes at $\phi_1 = 0$ and $\Phi = M$ but for $\phi_1 \neq 0$ we have $V > 0$ and the vev of Φ is $\Phi < M$. The fact that when the inflaton potential is included $\langle \Phi \rangle$ is smaller than M ($\Phi \simeq M/3$ for $\phi_1 \simeq 1$) can help to explain V_{Unif} and V_{Inf} in terms of a single scale (i.e. to express M in terms of the unification scale M_{gut}).

The conditions for successful inflation

$$(i) \frac{V_1}{V} = \frac{2}{\phi_1(1 + 16\phi_1^2)} \ll \sqrt{K_1^1} \quad (ii) \frac{V_{11}}{V} = \frac{2(1 - 48\phi_1^2)}{(\phi_1(1 + 16\phi_1^2))^2} \ll K_1^1 \quad (16)$$

for fields with non-canonical kinetic terms are clearly satisfied in the region $\phi_1 \simeq 1$. Inflation comes to an end when one of these inequalities is not satisfied. In our example this happens first for $\frac{V_{11}}{V} = K_1^1$ which gives $|\phi_1| \simeq 0.65$.

The number of e-folds is given by [10]

$$\begin{aligned} N &= - \int \left[\frac{V}{V'} (K_1^1 + K_2^1) \right] d\Phi_1 \\ N &= \int \frac{\phi_1(1 + 16\phi_1^2)}{Q^2} d\Phi_1 \end{aligned} \quad (17)$$

where we have taken the fields ϕ_i to be real fields, $\sqrt{K_1^1 + K_2^1} = \frac{\sqrt{2}}{Q}$ and we took the contribution of both fields since $\phi_1 = \phi_2$. Integrating eq.(17) one finds

$$N = \left[\frac{17}{4Q} + 2\log(Q/2) \right] |_{init} \quad (18)$$

where the subindex “*init*” refers to the initial value of the field ϕ_1 . From eq.(18) we see that a large number of e-folds requires $Q = 2 - 2|\phi_1|^2 \ll 1$, i.e. $|\phi_1|^2 \simeq 1$. For $Q \ll 1$ the number of e-folds can be approximated by the first term of eq.(18), i.e. $N = 17/4Q$.

Finally the fluctuations are given by [22]

$$\delta_H = \frac{1}{\pi\sqrt{75}} \sqrt{K_1^1 + K_1^2} \frac{V^{3/2}}{V_1} \quad (19)$$

where we have included the term $\sqrt{K_1^1 + K_1^2} = \frac{\sqrt{T_{r1}}}{Q}$ since the inflaton is not canonically normalized. Using eqs.(16), (18) (19) for $Q \ll 1$ the fluctuations can be written as

$$\delta_H = \frac{4N}{\pi\sqrt{150}} V^{1/2} \quad (20)$$

The field values where we calculate the fluctuations are determined by the horizon scale today: for a fluctuation emitted with a certain wavelength during inflation, one may calculate the wavelength that the fluctuation has today and compare this to the horizon distance (6000 Mpc) [21]. Indeed, during inflation a wave emitted at some value ϕ_1 increases its wavelength. In order to solve the horizon and flatness problems the number of e-folds of inflation is [22]

$$N = 62 + \ln \left(\frac{V_{Inf}^{1/4}}{10^{16} GeV} \right) + \ln \left(\frac{V_{Inf}^{1/4}}{V_{end}^{1/4}} \right) - \frac{1}{3} \ln \left(\frac{V_{Inf}^{1/4}}{\rho_{reheat}^{1/4}} \right) \quad (21)$$

where $\rho_{reheat} = \frac{\pi^2 g_*(T_R)}{30} T_R^4$ is the energy density after reheating, g_* the number of relativistic degrees of freedom (915/4 for the minimal supersymmetric standard model) and T_R the reheating temperature. The exact time of emission of a fluctuation with horizon size today, depends on the value of T_R , which is model dependent. To determine T_R we calculate the width of the inflaton field ϕ_1 . This field will decay into states Ξ of another sector of the theory by the interaction $\partial V / \partial \phi_1 W(\Xi)$ where the superpotential W should be replaced by the scalar components [7]. This interaction generates a trilinear coupling to the light fields of strength $\sim m_{\phi_1}$ giving a width $\Gamma_{\phi_1} \sim m_{\phi_1}^3 / (2\pi)^3$, where m_{ϕ_1} is the mass of the inflaton. The temperature associated with the radiation of the inflaton decay is [7]

$$T_R \sim \left(\frac{30}{\pi^2 g_*} \right)^{1/4} \Gamma_{\phi_1}^{1/2} \simeq 2.2 \times 10^{-2} m_{\phi_1}^{3/2} \quad (22)$$

and the mass of the inflaton is calculated at the end of inflation.

Solving eqs.(21), (22) and (20) for $\delta_H = 2.5 \times 10^{-5}$, as required by COBE [14] we obtain $N \simeq 56$, $T_R = 4.5 \times 10^8 GeV$, $m_{\phi_1} = 1 \times 10^{13} GeV$ and an inflation scale of $V^{1/4} \simeq 5 \times 10^{15} GeV$. Note that the reheating temperature is consistent with the bounds put by the relic abundance if the gravitino mass is of the order $1 TeV$ [7],[23].

The spectrum of fluctuations in this example is almost scale invariant. The spectral index is [22]

$$n = 1 + 2 \frac{1}{K_1^1 + K_1^2} \frac{V_{11}}{V} - \frac{1}{3(K_1^1 + K_1^2)} \left(\frac{V_1}{V} \right)^2 = 0.99 \quad (23)$$

where we have again taken into account for the non canonical kinetic term of ϕ_1 and included the contribution of ϕ_2 since $\phi_2 = \phi_1$. This spectral index corresponds to a slightly tilted

spectrum which has less power on galactic scales in a cold dark matter universe and therefore agrees better with the observations.

We have seen that a simple superstring model can lead to enough e-folds of inflation to solve the horizon and flatness problems and the density fluctuations of the inflaton field can be normalized to COBE once the dilaton field has been stabilized.

The stability of S is model independent (i.e. does not depend on the chiral superpotential P) and allows to have an inflationary potential at any scale. Note that the only condition needed is to have a positive potential but this is clearly no constraint since any inflationary potential must be positive anyway. Therefore, with an S -duality potential there is no longer a need to be concerned about the dynamics of the dilaton field and one can look for inflationary potentials in general supergravity models.

It is also interesting to note that with an S -dual potential we can have two or more stages of inflation. The first may occur at a large scale, as we have described so far, and depends on the scale of a spontaneously broken symmetry. This stage of inflation will solve the horizon and flatness problems and will give rise to the density fluctuations observed by COBE. An additional stage of inflation may occur below the supersymmetry breaking scale [10]. This later scale of inflation is welcomed to solve the Polonyi problem [28]. Of course, if the number of e-folds of inflation at the second stage is very large, then it will erase the original density fluctuations generated at the first stage and the resulting density fluctuation will be too small. Providing that this does not occur, even if we assume that there exist two stages of inflation, the spectrum of fluctuations that we predict is approximately a scale-invariant Harrison-Zeldovich one, and comes from the first stage of inflation. This differs from other models which use two stages of inflation [24] so as to reconcile the observed discrepancy between the COBE observations and the existing cold or hot dark matter models for structure formation [25]. However, this discrepancy may be explained, either by considering schemes where both cold and hot dark matter are present, or by taking into account the effect of additional sources of fluctuations. For example, we have shown that, unlike what was previously thought, under certain conditions, domain walls may enhance the standard cold dark matter spectrum without inducing unacceptable cosmic microwave background distortions. This occurs provided that either one of the minima of the potential of the scalar field ϕ is favoured [26], or the domain walls are unstable and annihilate after having induced fluctuations to the cold dark matter background [27]. Such solutions are predicted to exist in superstring models, therefore we believe that the overall picture that we have for inflation and structure formation in the framework of these theories is consistent.

Finally, let us comment on the possibility that for a fixed $P(\phi), P_m(\phi)$ the dilaton and/or moduli can work as inflaton fields. For S far away from its minimum ($S \gg 1$), the potential has an exponential behaviour due to the $\eta(S)$ term, i.e. $V_0 \simeq e^{\alpha S_r}$, with coefficient $\alpha = \pi/12$. Since this potential is not flat enough it does not lead to inflation. The same conclusion holds for the moduli fields $T_{2,3}$. Around the extremum $S = 1, e^{-\pi/6}$ the potential is flat enough, however inflation will not come to an end if P, P_m are constant, since in this case the potential will remain positive, even at the extremum of S . It is therefore unnatural to assume that $P(\phi), P_m(\phi)$ do not vary and we have to study their dynamics. In this

framework, suppose first that ϕ has a local minimum; then it is possible to find an "old" inflationary potential where the general minimum is obtained by vacuum tunneling from the local minimum. However, it is well known that such a solution leads to phenomenological problems (unless going to models of extended inflation). On the other hand, we can examine the case with a continuous variation w.r.t. ϕ . In this case, since the variation of ϕ will in general be larger than that of S around its extremum, the inflaton field will then correspond to ϕ and not to S .

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